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ABSTRACT

The need to consider the interaction of characteristics that affect students' learning is stressed. Levels of cognitive development as proposed by Piaget are presented first, with how they relate to teaching mathematics discussed. Then two cognitive styles, relating to the way an individual most efficiently processes information, are considered: impulsivity/reflection and field dependence/independence. Other facts are also noted: intelligence and motivation, modality preference, and preparation to learn. Three paper-and-pencil activities are included to aid the teacher in assessing the Piagetian tasks of displacement of volume and reversibility. (MNS)

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R & D INTERPRETATION SERVICE BULLETIN

**MATHEMATICS**

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# Individual Differences

Jean Sealey

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**QUESTION:** *What are some of the major differences in learning styles and levels of development among school students? How should these differences be dealt with in mathematics instruction?*

When we look for reasons for students' poor performance in school, we most often focus on deficits: "Johnny can't read." "Sue can't add two-digit numbers." We need to ask questions that focus on the student's strengths: "What does David know?" (or, "What can he do?") and "How does Karen learn?"

Psychologists have studied extensively the question of how students learn. As a result, we have a wealth of information about teacher behaviors, instructional strategies, and learner characteristics that affect students' learning. The interaction of these factors promotes varying degrees of learning. When a student fails to learn, we need to examine the *interaction* of characteristics rather than each characteristic in isolation. Specifically, we need to look at how a student is being taught relative to how that student learns best. Suppose, for example, that you have a student in your mathematics class who responds well when you present problems orally, but who performs poorly on written tests. You determine that the student can read, has no visual handicaps, and has the ability to perform. It is possible that the student is an auditory learner. For this student, information that is presented orally may be processed much more efficiently than information that is presented visually. The preference for one sensory modality is but one of several dimensions of individual differences that affect learning.

In this bulletin we discuss levels of cognitive development and differences in cognitive style as they relate to learning, and we suggest ways to deal with these differences in teaching mathematics.

## OVERVIEW

Students differ in many important ways. They differ in both their ability to learn and their motivation to learn. They differ also in *how* they learn. Piaget's observations of children give us information about movement through various levels of cognitive development — from egocentric, experience-based thought to logical, abstract thought. By considering the level of cognitive development and the cognitive style of a student, we can match the content and method of instruction to the student's needs. More information on how individual differences affect mathematics learning can be found in the *Research Within Reach* publications *Elementary School Mathematics* and *Secondary School Mathematics* developed by the Research and Development Interpretation Service (3, 4). The numbers in parentheses correspond to references at the end of the article.

## COGNITIVE DEVELOPMENT

JEAN PIAGET IS A name that most teachers know. Piaget, a Swiss psychologist, observed and studied children for over half a century. The theory that grew from his work has shaped modern thinking about the development of intelligence. Its greatest impact has been upon the teaching of mathematics and science in the elementary school (7). To better understand how different levels of cognitive development affect a student's ability to learn certain

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concepts, let's briefly review Piaget's theory.

Piaget described thought in terms of operations. Children are capable of increasingly complex and abstract operations as they mature; experience their world; learn from their parents, teachers, and other children; and seek to coordinate and regulate new knowledge. Until about age 6 or 7, children are *preoperational*. Their thinking is egocentric, or based on their own experiences. Between first grade and high school, children typically progress from concrete operations to formal operations.

Children in the *concrete operational stage*, from about 7 to 11-12 years of age, can manipulate objects and symbols. They can sort and classify. They can add, subtract, multiply, and divide. They are not, however, capable of dealing with abstractions, such as inferences and hypotheses, because their thought processes are still tied to their own concrete experiences.

During this stage, children achieve the concept of reversibility. This concept speeds up their progression toward formal operations. It enables children to cancel what they have observed and return to a previous thought. To determine if a child had attained reversibility, Piaget would give a child a piece of clay. He would then tell the child to break the clay into two pieces of the same size. Piaget would roll one piece into a long, thin sausage shape. Piaget would ask the child which piece was bigger. A child who was able to follow the transformation and then reverse it mentally to the original piece of clay would say the two pieces were the same size. The child had acquired reversibility.

As children move into adolescence, they begin to think beyond what is real and begin to deal with what is possible. This is the stage of *formal operations*. Now children can "operate on operations" or think about thinking (7). They can make inferences and hypothesize.

How does Piaget's theory relate to teaching mathematics? The developmental nature of mental operations suggests that teachers, particularly mathematics teachers, need to present concepts, select materials, and use methods of presentation that are appropriate for a student's level of cognitive development. This match begins with *determining* a student's level of cognitive development. Following Piaget's example, teachers can observe children's performance in specific situations to determine their level of thinking. Our insert describes some of the tasks used to determine children's cognitive level. It's in the form of a paper-and-pencil test that gives an indication of early formal thought.

More than a casual observation of students is required, however. For example, in one study all the students in a kindergarten class were able to count; some could count to 1,000. The teacher soon found that, although the students could count, none of them had the concept of numbers beyond three. When the students were asked to pick up four objects to match the number of objects in the teacher's hand, only one student was able to do so. The others picked up as many as 12 objects. The one student who was able to match correctly first counted to four on her own fingers and then matched the number of objects to the number of fingers she had counted. Ingenious though the method, the child demonstrated the need to anchor the problem to the concrete and to her own experience (1). Students, especially those with good verbal skills, may be able to name objects or concepts, just as these kindergartners could name numbers. It is important, then, to probe beyond a student's answer to determine if the student truly understands the concept.

After determining a student's cognitive level, teachers need to examine the textbook or other instructional materials. Are they appropriate for the student's level of development? It's not always safe to rely solely on the grade or age levels recommended by the publisher or manufacturer. Grade and age levels do not necessarily correspond to cognitive development levels (2). Students differ in the speed at which they develop their thinking skills. Although Piaget assigned approximate age levels to each stage he described, he recognized that students will not all reach a particular level at the same chronological age. Indeed, subsequent research has shown that many children do not attain the stage of formal operations by 12 or 14 as Piaget orginally stated. Moreover, when students begin to exhibit an ability to think formally, we cannot assume that they can think formally on all tasks (3). So, teachers need to determine what types of thinking are required to master the materials.

Knowing a student's level of cognitive development also enables teachers to choose the most effective methods of instruction. Students in the concrete operations stage need opportunities to manipulate objects. Number concepts can be presented with a variety of objects including base-ten blocks, Cuisenaire rods, and graphic representations. Multiple representations should enable students to recognize the mathematical commonalities, but if they don't, teachers can direct attention to them (3).

## COGNITIVE STYLES

COGNITIVE STYLE REFERS TO THE WAY in which an individual most efficiently processes information. Researchers have classified cognitive styles along several dimensions. The two that we will discuss here have a direct bearing on the learning and teaching of mathematics. Keep in mind that all individuals possess these characteristics in varying degrees, and that an individual can be classified on more than one dimension.

*Impulsivity and reflection.* Any teacher of mathematics has observed that students react differently when presented with a problem. Some will repeat the problem, ask for clarification, and, in general, take longer to respond. Others will seem compelled to answer immediately. This difference in style has been described as the dimension of reflection/impulsivity. It has been suggested that reflective students are more anxious about making mistakes and that impulsive students are less inhibited and less cautious.

Impulsive students may fare better in certain types of mathematics activities, like basic skills drills, where speed of response is stressed (3). However, studies examining the effects of these two styles suggest that, in general, reflective students perform better in mathematics than their impulsive classmates (8). When students tend to respond at either end of the impulsive-reflective continuum, they may require special help in the classroom. Different strategies may be used to facilitate learning for these two groups of students:

- For an overly reflective student, try to alleviate anxiety. Pose questions in a nonthreatening way. Let the student know it is alright to make mistakes. Encourage the student to guess, and then help the student find the correct answer if the guess is incorrect.
- For an extremely impulsive student, suggest the possibility of errors when the problem is not given enough thought. Encourage the student to take more time to consider the problem. Emphasize the quality and accuracy of answers rather than the speed of responding.

*Field dependence/field independence.* Another dimension of cognitive style that differentiates individuals is the degree to which they rely on the surrounding background, depend on authority, and prefer social contacts. Field-dependent individuals rank high in these characteristics; field-independent individuals do not (8). For example, a student who is field-dependent has difficulty extracting information embedded in text, will require more directions from the teacher, and prefers to be with people. A field-independent student, on the other hand, can ignore distracting cues to extract relevant information, can work independently, and can remain socially detached (5, 8). While we know that individuals vary on this dimension, researchers have identified at least two groups of students who tend to fall at the field-dependent end of the spectrum: disadvantaged children (5) and young children (5, 8). Since field-dependent students generally have more difficulty in mathematics, let's look at some ways to help these students.

1. Provide opportunities for students to practice extracting information embedded in a background. For example, give word problems with irrelevant information and ask them to determine what is needed to solve the problem. Give students word or picture puzzles that require them to extract a word or an object. The embedded picture puzzles, such as those found in *Highlights* magazine, are good for younger children, and the "word seek" puzzles found in commercial puzzle books can be used with older students. You can also have students construct such puzzles themselves giving them added insight into the nature of the problem.
2. Encourage and reward independent activities. Gradually reduce the amount of assistance you give the student. As you do this, be less critical of errors as long as the student is making the effort independently.
3. Ask more open-ended questions and set up learning situations in which students can discover for themselves the correct answer.

## OTHER FACTORS

COGNITIVE DEVELOPMENT AND cognitive style are only two ways of describing how individuals are different. Among others most relevant to learning are intelligence and motivation, sensory modality preference, and preparation to learn.

Students, of course, differ in their measured intelligence and in their motivation to learn. These two factors are mentioned together because of the effect one can have on the other. Students with lower measured intelligence may outperform students with greater ability if they are sufficiently motivated. A well-motivated student will work harder and longer to achieve success. Teachers can't substantially change a student's intelligence, but they can motivate students. Therefore, we suggest that it is more beneficial to focus on motivating students than it is to categorize them by ability level. Because success itself is motivating, start by giving students tasks that they can complete successfully. Then recognize and praise their accomplishments (6).

Students also differ in the sensory modality through which they best process information. Students may show a definite preference for processing information visually, aurally, or tactilely. These preferences can be observed in the student's behavior. Does the student seem to be able to learn material from lectures without taking notes? Does the student learn more readily from reading? Does the student seem to require that information be presented in several ways before it can be retained? Identify and observe behaviors that will give you an indication of a student's sensory modality preference. Use your awareness of the student's preference to present information in ways that will help the student learn.

When students enter your classroom, they differ in their preparation for learning mathematics. Two

students may enter the first grade together and be in the same class throughout elementary school, yet when they enter your 7th-grade mathematics class, they will have a different knowledge of the prerequisites. One may have been sick when a basic concept was presented; one may have been daydreaming. When you consider that most of your students have been exposed to different teachers, different textbooks, and different approaches to mathematics, it is easy to see that all students are not equally prepared to receive instruction. For every student who enters your classroom, it is important that you determine if topics have not been learned, or if misinformation has been acquired.

## SUMMARY

WE'VE DISCUSSED INDIVIDUAL differences in level of cognitive development and cognitive style in some detail. We've touched briefly on some other differences that affect learning, such as intelligence and motivation, sensory preferences, and preparation. Obviously we haven't exhausted the list of dimensions on which individuals differ. Nor was that our intention. Our purpose here is to help you think about ways in which students are different and how these differences affect learning, particularly in mathematics. When you observe your students, be alert to these differences. When you design your instruction, capitalize on a student's strengths and accomodate those traits that may inhibit learning.

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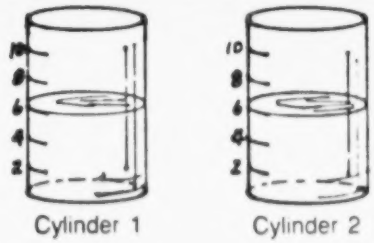
**TASK ONE: Displacement of Volume (2)**

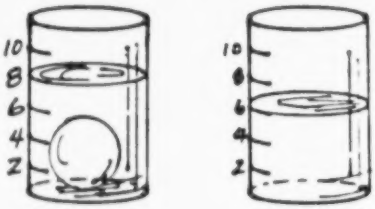
• Here are drawings of two vertical tubes (cylinders) that are filled to the same mark with water. The cylinders are identical in shape and size.




• Here are two marbles, one made of steel and one made of glass. Both marbles have the same volume (that is, they are both the same size). The steel marble is heavier. ►

The steel marble is heavier than the glass one, but both marbles will sink if placed in the cylinders. We are going to put one marble in each of the cylinders.

• After we have put the glass marble in Cylinder 1, the cylinders and their contents look like this:

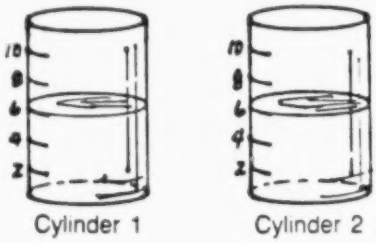



1. Now if we add the steel marble to Cylinder 2, what will happen to the level of water in Cylinder 2? (Tell whether it will rise, fall, or stay the same, and if it rises or falls, tell what the final water level will be in Cylinder 2.)

2. Explain why your experiment had the result it did.

**TASK TWO: Displacement of Volume**

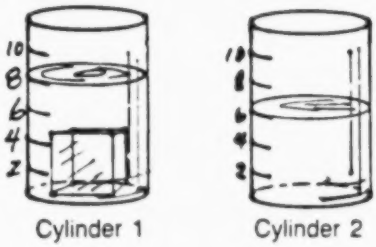
• Here are drawings of two vertical tubes (cylinders) that are filled to the same mark with water. The cylinders are identical in shape and size.




• Here are a cube and a ball. They are both made of glass. They both weigh the same. ►

They will both sink when placed in the cylinders. We are going to put the cube in one cylinder and the ball in the other cylinder.

• After we have put the cube in Cylinder 1, the cylinders and their contents look like this:




3. Now if we add the ball to Cylinder 2, what will happen to the level of water in Cylinder 2? (Tell whether it will rise, fall, or stay the same, and if it rises, tell what the final water level will be in Cylinder 2.)

4. Explain why your experiment had the result it did.

**TASK THREE: Reversibility**

• Here is a drawing of a lump of clay. ►

• We are going to divide the clay into two equal pieces, that is, both pieces will be the same size.

• After we have divided the clay into two equal pieces, we are going to roll piece A into a ball and piece B into a sausage. ►

5. Which of the following statements is true:
   A is bigger than B.
   B is bigger than A.
   A and B are the same size.

6. Explain your answer.

# MATHEMATICS

## Paper and Pencil Piagetian Tasks

Piaget's theory of cognitive development is particularly relevant to mathematics. Learning abstract mathematical concepts requires what Piaget described as formal operations: the ability to make inferences and formulate hypotheses. Not all children attain the level of formal operations at the same chronological age. How can we determine, then, if a student has begun to think formally? We could interview each student, as Piaget did, presenting situations and asking the child to explain the reasoning used to arrive at an answer. The amount of time required for this procedure, however, makes it impractical for the classroom teacher.

Researchers have devised paper and pencil versions of Piagetian tasks. One such test (the first one described on the back of this page) uses the displacement of volume to indicate early formal reasoning in children (2). The researchers administered both the paper-and-pencil test and the interview procedure to 118 eleventh-grade students, 46 sixth-grade students, and 25 fifth-grade students. The results of the paper-and-pencil version were compared to the results of the interview technique. The researchers reported that the two methods yielded the same classification for 94% of the test subjects. That is, this paper-and-pencil test appears to be a valid indicator of early formal operations for most students.

Please keep in mind that this is a screening device, and that it will not be accurate 100% of the time.

Other Piagetian tasks may be adapted to paper-and-pencil tests. We've included two examples that we adapted to show you other ways to test for early formal operations. None of these tests can be used to establish unequivocally the presence or absence of formal thought. Remember to use other evidence, such as the student's explanation of answers or the student's behavior in other situations.

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Solutions: Explanations similar to these indicate early formal thought.

1. Rise, to 8.
2. The two balls have the same volume and will displace the same volume of water.
3. Rise, to 8.
4. The ball and the cube are same material and same weight, therefore are equal in volume and will displace the same volume of water.
5. Both the same.
6. They were the same size to begin with.